UNIVERSITY OF MANITOBA
Term Test 2C
COURSE: MATH 1500
DATE & TIME: November 1, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 5 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 45 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	12	
2	7	
3	5	
4	4	
5	8	
6	9	
Total:	45	

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1. Find the derivative of each of the following functions. DO NOT SIMPLIFY YOUR ANSWER AFTER YOU EVALUATE THE DERIVATIVE.

[4] (a) $f(x) = e^{5x} - 3x^{2/3} + (8\pi)^{-4} + \dfrac{7}{x}$.

Solution: $f'(x) = 5e^{5x} - 2x^{-\frac{1}{3}} + 0 - 7x^{-2}$.

[4] (b) $g(u) = 4^u(\sin u + 3\ln u)$.

Solution: $g'(u) = 4^u \ln 4(\sin u + 3\ln u) + 4^u\left(\cos u + \dfrac{3}{u}\right)$.

[4] (c) $h(z) = \dfrac{(3z-5)^4}{2+4z^3}$.

Solution:

$$\begin{aligned} h'(z) &= \frac{((3z-5)^4)'\,(2+4z^3) - (3z-5)^4(2+4z^3)'}{(2+4z^3)^2} \\ &= \frac{(4)(3z-5)^3(3)\,(2+4z^3) - (3z-5)^4(12z^2)}{(2+4z^3)^2}. \end{aligned}$$

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[7] 2. Find the derivative of the following function. Express your final answer as a function of only the variable x. DO NOT SIMPLIFY YOUR ANSWER.

$$f(x) = (x-5)^{-\sin x}.$$

Solution: Let $y = (x-5)^{-\sin x}$. Using logarithmic differentiation, we first take the logarithm of both sides and apply the properties of logarithms before taking the derivative.

$$\begin{aligned} \ln y &= \ln(x-5)^{-\sin x} \\ \ln y &= (-\sin x)\ln(x-5). \end{aligned}$$

Next, we differentiate both sides with respect to x and then solve for $\frac{dy}{dx}$:

$$\begin{aligned} \frac{1}{y}\frac{dy}{dx} &= (-\cos x)\ln(x-5) + (-\sin x)\left(\frac{1}{x-5}\right) \\ \frac{dy}{dx} &= y\left((-\cos x)\ln(x-5) - \frac{\sin x}{x-5}\right) \\ \frac{dy}{dx} &= (x-5)^{-\sin x}\left((-\cos x)\ln(x-5) - \frac{\sin x}{x-5}\right). \end{aligned}$$

3. A particle is moving along the x-axis, and its displacement in meters after t seconds is given by the function $x(t) = t^2 - 7t + 5$.

[2] (a) What are the velocity and acceleration of the particle at any time?

Solution:
Velocity, $v(t) = x'(t) = 2t - 7$ m/s.
Acceleration, $a(t) = x''(t) = 2$ m/s^2.

[3] (b) When is the speed of the particle equal to 3 m/s?

Solution:
The speed of the particle is equal to 3 m/s when,
$|v| = 3 \Longrightarrow |2t - 7| = 3 \Longrightarrow 2t - 7 = 3 \quad \text{or} \quad 2t - 7 = -3 \Longrightarrow t = 5 \quad \text{or} \quad t = 2.$
So, the speed of the particle is equal to 3 m/s when $t = 2$ s and $t = 5$ s.

4. Calculate each limit below. If a limit does not exist, explain why. Show all work. Writing an answer with no justification may not yield any marks.

[1] (a) $\lim\limits_{x \to 0^+} 2\log_7 x$.

Solution: Since $\lim\limits_{x \to 0^+} \log_7 x = -\infty$ then $\lim\limits_{x \to 0^+} 2\log_7 x = -\infty$.

[3] (b) $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{\sin(9x)}$.

Solution:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin(5x)}{\sin(9x)} &= \lim_{x \to 0} \frac{\sin(5x)}{5x} \left(\frac{9x}{\sin(9x)} \right) \left(\frac{5x}{9x} \right) \\
&= \lim_{x \to 0} \frac{\sin(5x)}{5x} \lim_{x \to 0} \left(\frac{9x}{\sin(9x)} \right) \lim_{x \to 0} \left(\frac{5x}{9x} \right) \\
&= (1)(1)(5/9) = \frac{5}{9}.
\end{aligned}$$

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[8] 5. Find an equation of the tangent to the curve defined implicitly by

$$y^3 + xy = x^2 + 1$$

at the point $(1, 1)$.

Solution:

Differentiating the given equation with respect to x, we get

$$\begin{aligned} \frac{d}{dx}(y^3 + xy) &= \frac{d}{dx}(x^2 + 1) \\ 3y^2\frac{dy}{dx} + y + x\frac{dy}{dx} &= 2x \\ \frac{dy}{dx} &= \frac{2x - y}{3y^2 + x} \end{aligned}$$

Let m be the value of $\frac{dy}{dx}$ when $x = 1$ and $y = 1$, then $m = \frac{1}{4}$.
Therefore, an equation of the tangent is

$$y - 1 = \frac{1}{4}(x - 1).$$

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[9] 6. A rectangular box is designed so that its length is always exactly three times its height. At a particular moment, the box has length 12 cm, width 6 cm and height 4 cm. At this moment, the length is increasing at 2 cm/s. What is the rate of change of width at that moment, if the volume of the box is increasing at 240 cm^3/s? State your final answer in a complete sentence.

Solution: Let l be the length, w the width and h the height. (The students should draw a diagram and label it)
We have $l = 3h$ at all times.
Thus, the volume V is given by

$$V = lwh = \frac{wl^2}{3}.$$

We know that $\frac{dV}{dt} = 240$ cm^3/s and $\frac{dl}{dt} = 2$ cm/s.
We want to find $\frac{dw}{dt}$ when $l = 12$, $w = 6$, and $h = 4$.
Differentiating gives

$$\frac{dV}{dt} = \frac{2}{3}wl\frac{dl}{dt} + \frac{l^2}{3}\frac{dw}{dt}.$$

Replacing by the given values,

$$240 = \frac{2}{3}(6)(12)2 + \frac{12^2}{3}\frac{dw}{dt} \Longrightarrow \frac{240 - 96}{48} = \frac{dw}{dt} \Longrightarrow \frac{dw}{dt} = 3.$$

Therefore, at the particular moment in question, the width of the box is increasing at the rate of 3 cm/s.

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